MATH 20D: Differential Equations Spring 2023
Homework 6

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Remember to list the sources you used when completing the assignment.
Below *NSS* is used to reference the text *Fundamentals of Differential Equations (9th edition) by Nagle, Saff, Snider*

Question (1). *Solve the initial value problems using the method of Laplace transforms*
(a) $y'' - y' - 2y = 9$; $y(0) = -2$, $y'(0) = 5$.
(b) $y'' + 6y' + 5y = 12e^t$; $y(0) = -1$, $y'(0) = 7$.
(c) $w'' + w = t^2 + 2$; $w(0) = 1$, $w'(0) = -1$.
(d) $y'' - 7y' + 10y = 9\cos(t) + 7\sin(t)$; $y(0) = 5$, $y'(0) = -4$.
(e) $y'' - 4y = 4t - 8e^{-2t}$; $y(0) = 0$, $y'(0) = 5$.

Question (2). *Sketch the graph of the given functions and determine their Laplace transform*

   (a) $f(t) = (t-1)^2 u(t-1)$, $t \geq 0$,   **(b)** $f(t) = t^2 u(t-2)$, $t \geq 0$,

   (c) $f(t) = u(t-1) - u(t-4)$, $t \geq 0$,   **(d)** $f(t) = t^2 \Pi_{2,4}(t)$,

   (e) $f(t) = tu(t-1)$, $t \geq 0$,   **(f)** $f(t) = u(t - \pi/2)\sin(t)$, $t \geq 0$

Question (3). *Determine the inverse Laplace transforms of the given functions*

   (a) $F(s) = \frac{e^{-3s}}{s^2}$,   **(b)** $F(s) = \frac{e^{-3s}}{s^2+9}$,

   (c) $F(s) = \frac{e^{-s}}{s^2+4}$,   **(d)** $F(s) = \frac{e^{-s}(3s^2 - s + 2)}{(s-1)(s^2+1)}$

Question (4). *Solve the initial value problems using the method of Laplace transforms*
(a) $w'' + w = \Pi_{2,4}(t)$; $w(0) = 1$, $w'(0) = 0$.
(b) $y'' + y = 3\sin(2t) - 3\sin(2t)u(t - 2\pi)$; $y(0) = 1$, $y'(0) = -2$.
(c) $y'' + 4y' + 4y = u(t - \pi) - u(t - 2\pi)$; $y(0) = 0$, $y'(0) = 0$.
(d) $y'' + 5y' + 6y = tu(t - 2)$; $y(0) = 0$, $y'(0) = 1$.

(e) $y'' + 2y' + 10y = g(t)$; $y(0) = -1$, $y'(0) = 0$ *where* $g(t) = \begin{cases} 10, & 0 \leq t \leq 10, \\ 20, & 10 < t < 20 \\ 0, & 20 < t \end{cases}$

(f) $y'' + 3y' + 2y = g(t)$; $y(0) = 2$, $y'(0) = -1$ *where* $g(t) = \begin{cases} e^{-t}, & 0 \leq t \leq 3, \\ 1, & 3 < t. \end{cases}$

Question (5). *At 3:00pm the mixing tank holds 500L of a brine solution with a salt concentration of* $0.02kg/L$.

12 L/min
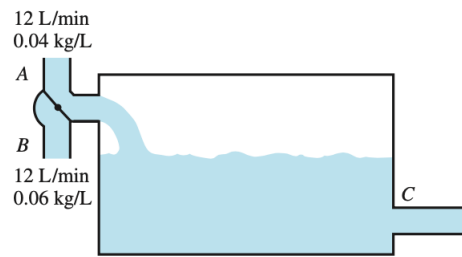0.04 kg/L

A

B

12 L/min
0.06 kg/L

C

Figure 7.18 Mixing tank

Between 3:00pm and 3:10pm, valve A is open releasing a brine solution containing 0.04 kilograms of salt per liter at a rate 12L/min into the tank. At 3:10pm valve B is switched in and releases a brine solution containg 0.06 kilograms of salt per liter at a rate 12L/min into the tank. The exit valve C removes 12L/min, thereby keeping the volume constant for the duration of the mixing. Using the method of Laplace transform, find the concentration of salt in the tank as a function of time.